Control of systematic uncertainties in the storage ring search for an electric dipole moment by measuring the electric quadrupole moment

Andrzej Magiera*

Institute of Physics, Jagiellonian University, 30-348 Kraków, Poland

(Received 7 August 2017; published 27 September 2017)

Measurements of electric dipole moment (EDM) for light hadrons with use of a storage ring have been proposed. The expected effect is very small, therefore various subtle effects need to be considered. In particular, interaction of particle's magnetic dipole moment and electric quadrupole moment with electromagnetic field gradients can produce an effect of a similar order of magnitude as that expected for EDM. This paper describes a very promising method employing an rf Wien filter, allowing to disentangle that contribution from the genuine EDM effect. It is shown that both these effects could be separated by the proper setting of the rf Wien filter frequency and phase. In the EDM measurement the magnitude of systematic uncertainties plays a key role and they should be under strict control. It is shown that particles' interaction with field gradients offers also the possibility to estimate global systematic uncertainties with the precision necessary for an EDM measurement with the planned accuracy.

DOI: 10.1103/PhysRevAccelBeams.20.094001

I. INTRODUCTION

A storage ring can be used to search for an electric dipole moment (EDM) of charged particles [1]. The main idea of such a measurement is to observe build-up of vertical polarization induced by an EDM for the initially horizontally polarized beam. Various measurement scenarios are considered. It has been proposed [2] to use longitudinally polarized protons stored in a purely electrostatic ring. With a proper choice of the electric field, the $(g-2)$ precession vanishes and the horizontal spin is always aligned with the momentum vector—therefore it is called "frozen-spin" method. With the proper combination of electric and magnetic fields this method could be applied also for deuterons where the $(g-2)$ anomaly is negative. Dedicated experiments utilizing this method have been proposed at BNL [3] with the aimed EDM precision of 10^{-29} e cm for protons and deuterons. For particles with small anomalous magnetic moment, like deuteron, the "quasi-frozen-spin" method could be applied [4,5]. In this method the spin rotates in the horizontal plane, which reduces the EDM signal by a few percent. The resonance method for EDM measurements in storage rings [6] was also proposed. The idea is to use oscillating electric field in resonance with planar spin precession. In this case the nonvanishing EDM induces a growing vertical polarization. In the "partially-frozen-spin" method an rf Wien filter is included in a storage ring dedicated to EDM measurement [7]. It was shown that for the initially horizontally polarized particles with the proper choice of rf Wien filter frequency and phase, the EDM produces vertical polarization linearly growing with time.

The JEDI Collaboration (Jülich Electric Dipole Moments Investigations) [8] has been formed to demonstrate the feasibility of the EDM measurement with use of a storage ring and to perform the necessary developments towards the design of a dedicated storage ring [9]. The method with an rf Wien will be utilized in the planned precursor experiment at the COSY synchrotron [10], therefore the present paper focuses on various aspects of this method.

While with the most sensitive methods the statistical accuracy allowing to reach a sensitivity of 10^{-29} e cm for EDM could be reached in a few months of measurement, the major issue is a control on the systematic uncertainties. Various sources of systematic error were discussed in Ref. [1]. The Wien filter misaligment as a source of systematic uncertainty is discussed in Ref. [7] as an effect specific for "partially-frozen-spin" method. More detailed discussion of systematics in various EDM measurement scenarios is presented in Refs. [11,12]. As a major source of systematic error the rotation of the dipole magnets was identified. This type of ring imperfections leads to unwanted spin rotations that may mimic the EDM effect. Recently a novel method to study the systematic effects of the field imperfection was developed [13], allowing us to determine the angular orientation of the stable spin axis with high accuracy. Depending on the proposed technique of the measurement various methods to verify the

*andrzej.magiera@uj.edu.pl

systematics have been proposed, none of them fully satisfactory. Here a new method of controlling the systematics in the EDM measurement is proposed, requiring only minor changes in the experimental conditions.

In any storage ring that could be used for the EDM measurement the electromagnetic field gradients are naturally present. For a particle possessing a magnetic dipole moment (MDM) and/or electric quadrupole moment (EQM), additional spin precession occurs due to their interaction with field gradients. The values of MDM for proton and deuteron are known with the accuracy of 10^{-8} [14] and EQM for deuteron is known with the accuracy of 10^{-3} ($\mathrm{EQM}_d = 0.28590(30)$ e cm^2 [15,16]). This accuracy is sufficient to make use of the MDM and EQM interaction with field gradients to control systematic uncertainties in the EDM measurement to the level much lower than necessary to reach the 10^{-29} e cm sensitivity. Reproducing the known values of MDM and EQM in the storage ring measurement would demonstrate that all the systematics is well under control. In the present paper the field gradients effect will be always compared to the goal accuracy of 10^{-29} e cm for the EDM determination.

In Sec. II the equations of motion and spin precession generalized by MDM and EQM interaction with field gradients are discussed. Section III is devoted to analytical solution of the spin precession equations for deuterons circulating in a storage ring equipped with rf Wien filter. Numerical calculations for EDM and EQM effects for a simple storage ring are given in Sec. IV and a discussion of the results is presented in Sec. V.

II. EQUATIONS OF MOTION AND SPIN PRECESSION IN NONUNIFORM FIELDS

In the description of particle's motion in storage rings a standard Lorentz force is used, and for the description of the spin precession the Bargmann-Michel-Telegdi (BMT) equation [17] is applied. These equations are valid in homogeneous fields or when the interaction of MDM and EQM with the field derivatives can be neglected. This approach is sufficient for the description of particle's motion and spin precession induced by MDM to the first order. However, when considering the very small effect of spin precession induced by the EDM interaction, the second order terms containing electromagnetic field gradients must be considered.

These second-order effects induce additional interaction of the MDM with field gradients, changing therefore the equation of motion and, more importantly, modifying the spin precession equations. Further additional terms need to be introduced when considering particles possessing a nonvanishing electric quadrupole moment (as e.g. deuteron), which also interacts with electromagnetic field derivatives. The extension of the BMT equation by these additional terms was shown by Good [18], and later on different derivations were presented in [19,20]. In Ref. [20] the BMT equation was further extended considering the EDM effects and the recent derivation dedicated to the EDM problem can be found in Ref. [21].

The part of the spin precession equation corresponding to MDM and EQM interaction with field gradients in Ref. [18] was derived in terms of classical electrodynamics. For particles with spin larger than $1/2$ this result is inconsistent with the quantum mechanical approach of Refs. [22,23]. While it is not a place to discuss this discrepancy, the spin motion equation from Ref. [18] is used in a present paper. However, it should be mentioned that the final result is by a factor of γ^2 smaller when using the equation from Refs. [22,23].

A. Equation of motion

The equation of motion supplemented with MDM and EQM interaction with field derivatives has the form [18]

$$mc\frac{d(\gamma\vec{\beta})}{dt} = e\vec{E} + ec\vec{\beta}\times\vec{B} + g\frac{e\hbar}{2mc}I\gamma\left[\vec{\nabla} + \vec{\beta}\times(\vec{\beta}\times\vec{\nabla}) + \vec{\beta}\frac{\partial}{c\partial t}\right]\left\{\vec{s}\cdot\left[(\gamma-1)c\vec{B} - \frac{\gamma^2 c}{\gamma+1}(\vec{\beta}\cdot\vec{B})\vec{\beta} - \gamma\vec{\beta}\times\vec{E}\right]\right\}, \quad (1)$$

where I is a particle spin and $\vec{s}$ is a unit spin vector. The first two terms in Eq. (1) correspond to Lorentz force, and the third term describes the MDM interaction with field derivatives. The field gradients in this equation occur due to the $\vec{\nabla}$ operator acting on $\vec{B}$ and $\vec{E}$ fields. For time-dependent fields there is an additional term with the time derivative $\partial/\partial t$. The equation of motion does not contain terms with EQM interaction with field derivatives.

Checking the coefficients at the Lorentz force and the third term of Eq. (1) it is seen that the effect of the field gradients is by 16 orders of magnitude smaller than that for the Lorentz force. Therefore the effects of field gradients in the equation of motion can be usually neglected. In some specific cases, however, they become important, especially when dealing with measurements of very small effects, such as those induced by EDM. In particular it was shown [24] that due to the Stern-Gerlach effect a stored ion beam can be polarized by spatial separation of particles with different spin projections.

B. Spin precession equation

The BMT spin precession equation extended by MDM and EQM interaction with field derivatives [18] and including the terms generated by EDM [21] reads

$$\begin{aligned}\frac{d\vec{s}}{dt} = &\frac{e}{mc}\vec{s}\times\left[\left(\frac{g}{2}-1+\frac{1}{\gamma}\right)c\vec{B}-\left(\frac{g}{2}-1\right)\frac{\gamma c}{\gamma+1}(\vec{\beta}\cdot\vec{B})\vec{\beta}-\left(\frac{g}{2}-\frac{\gamma}{\gamma+1}\right)\vec{\beta}\times\vec{E}\right]\\&+\frac{D}{\hbar}\vec{s}\times\left[\vec{E}-\frac{\gamma}{\gamma+1}(\vec{\beta}\cdot\vec{E})\vec{\beta}+c\vec{\beta}\times\vec{B}\right]\\&+\frac{geI\hbar}{2m^2c^2}\frac{\gamma}{\gamma+1}[\vec{s}\times(\vec{\beta}\times\vec{\nabla})]\left\{\vec{s}\cdot\left[c\vec{B}-\frac{\gamma c}{\gamma+1}(\vec{\beta}\cdot\vec{B})\vec{\beta}-\vec{\beta}\times\vec{E}\right]\right\}\\&+\frac{Q\gamma}{\hbar(2I-1)}\left\{\vec{s}\cdot\left[\vec{\nabla}+\frac{\gamma}{\gamma+1}\vec{\beta}\times(\vec{\beta}\times\vec{\nabla})+\vec{\beta}\frac{\partial}{c\partial t}\right]\right\}\left\{\vec{s}\times\left[\vec{E}-\frac{\gamma}{\gamma+1}(\vec{\beta}\cdot\vec{E})\vec{\beta}+c\vec{\beta}\times\vec{B}\right]\right\},\end{aligned} \tag{2}$$

where D is a value of the electric dipole moment and Q is a value of the electric quadrupole moment. The first term in Eq. (2) corresponds to the standard BMT equation, the second term occurs due to the EDM effect, the third term describes the MDM interaction with field gradients and the fourth term is due to the EQM interaction with field derivatives. For a particle with spin $1/2$ (e.g. for proton) the electric quadrupole moment is equal 0, hence the spin precession equation reduces to the first three terms only.

The coefficients in the last three terms of Eq. (2) for particles for which the EDM measurements are planned (proton and deuteron) are of the same order of magnitude. This means that to reach the aimed precision of $D = 10^{-29}$ e cm in the planned EDM measurement one cannot neglect the effects of MDM and EQM interaction with field derivatives. Additionally, for realistic experimental conditions, the term with time derivative $\partial/\partial t$ is much smaller than terms with field gradients and therefore can be neglected. Remaining terms are by 15-16 orders of magnitude smaller than the leading first term corresponding to standard BMT equation.

III. SOLUTION OF SPIN PRECESSION EQUATION WITHIN A SIMPLE MODEL

The spin precession equation (2) can be solved numerically together with the equation of motion (1) using one of the existing tracking codes. However, presently none of the existing tracking codes provides the information on the spin precession in the presence of the field gradients. Inclusion of their effect on spin precession would allow to obtain an exact solution, provided the field spatial distributions are known with sufficient precision and a sufficient numerical accuracy can be reached, which would require large effort. Nevertheless an analytic solution, even within a simplified model, provides greater insight into results obtained with numerical solution. In the present paper such an analytic solution for a specific case was found using the *Mathematica* software [25].

In Ref. [7] a simple model for "partially-frozen-spin" method proposed for the EDM measurement was presented. In this model the particles circulate in the homogeneous static main field. In addition, there is an rf Wien filter uniformly distributed along the particles trajectory. Initially the particles are polarized horizontally in the plane of the trajectory. For the particles moving on the reference orbit an approximate solution of spin precession equation was presented for the electric and magnetic field of the Wien filter oscillating harmonically with the frequency equal to spin precession frequency. In this case for the particles with initial horizontal polarization, the EDM manifests itself as vertical polarization linearly increasing with time.

In the following, the simple model presented in Ref. [7] was extended by considering an rf Wien filter covering only a small part of particles orbit in a purely magnetic ring. Moreover, realistic distribution of magnetic field including field gradients was considered. This allows to identify properly all resonance frequencies of the rf Wien filter for which EDM shows up as a linear grow-up with time of vertical polarization. For a realistic field distribution in a natural way the field gradients are present, which allows to analyze the field gradients effects. To the best of our knowledge no such analysis has been performed and published before. Results of such analysis performed by author were presented on JEDI collaboration meetings [26]. While generally all field gradients should be considered, here the analysis is limited only to the derivative $\partial\vec{B}/\partial z$ of a fringe field of dipole magnets. For the particles moving on a reference orbit the presence of these gradients cannot be avoided due to the finite size of the magnets. Then in Eq. (2) for the particles with initial horizontal polarization the only term with field derivatives which remains is the one with EQM.

A pure magnetic ring with only vertical $\vec{B}$ field is considered for simplicity without loss of generality. The standard coordinate system is used with $\hat{x}$ pointing radially, $\hat{y}$ perpendicular to the trajectory plane and $\hat{z}$ along the particle momentum. With the orbital (revolution) frequency ω_o the spin precession frequency in the main field is $\omega_s = \gamma\omega_o(g-2)/2$. The main magnetic field has only vertical component B_0, the Wien filter magnetic field has only vertical component B_{0w} and radial electric field component E_{0w}. The amplitudes of the Wien filter fields are related by $E_{0w} = \beta c B_{0w}$, and Wien filter fields oscillate harmonically with frequency ω. The shape of bending dipoles field distribution and their gradients are represented by Fourier series:

$$B(t) = \frac{R_0}{2} + \sum_{j=1}^{\infty} R_j \cos j\omega_o t, \tag{3a}$$

$$G(t) = \frac{\beta c}{\omega_o}\frac{dB}{dz}(t) = -\sum_{j=1}^{\infty} jR_j \sin j\omega_o t. \tag{3b}$$

The particle moving on the reference orbit will pass the Wien filter limited in space with the orbital frequency ω_o. Therefore it is convenient to represent the shape of the Wien filter fields by the Fourier series

$$B_w(t) = \frac{E_w(t)}{\beta c} = \left[\frac{W_0}{2} + \sum_{m=1}^{\infty} W_m \sin(m\omega_o t + \varphi_m)\right] \cos(\omega t + \phi), \tag{4}$$

where ϕ is a phase of Wien filter field at time $t = 0$ and the Fourier coefficients W_m are calculated using a function describing Wien filter distribution along particle trajectory.

The following definitions are used:

$$\omega_W = \frac{eg}{2m\gamma^2} B_{0w} \tag{5}$$

$$\omega_e = \frac{D\beta c}{\hbar} B_0 \qquad \omega_g = \frac{Q\gamma}{\hbar}\omega_o B_0 \tag{6}$$

where ω_W—spin precession frequency due to Wien filter, ω_e—spin precession frequency due to EDM and ω_g—spin precession frequency due to fringe field of dipole magnets in the ring. There is no EQM interaction with the Wien filter gradients since their effect cancels due to relation between electric and magnetic field of the Wien filter.

With all these definitions the spin precession equations have the following form:

$$\frac{ds_x}{dt} = \Omega(t) s_z(t) \tag{7a}$$

$$\frac{ds_y}{dt} = -\omega_e B(t) s_z(t) + \omega_g G(t) s_z^2(t) \tag{7b}$$

$$\frac{ds_z}{dt} = -\Omega(t) s_x(t) + \omega_e B(t) s_z(t) + \omega_g G(t) s_z^2(t), \tag{7c}$$

where $\Omega(t)$, describing precession of the longitudinal spin component, is given by Eq. (8)

$$\Omega(t) = \omega_s\left(1 + \frac{2}{R_0}\sum_{n=1}^{\infty} R_n \cos n\omega_o t\right) - \omega_W\left(\frac{W_0}{2} + \sum_{m=0}^{\infty} W_m \sin(m\omega_o t + \varphi_m)\right)\cos(\omega t + \phi). \tag{8}$$

Since ω_e and ω_g are several orders of magnitude smaller than other terms, they can be neglected in Eq. (7c). With this approximation only Eqs. (7a) and (7c) are coupled and this set of equations can be solved analytically. The initial conditions for time $t = 0$ are assumed to be $s_x(0) = 0$ and $s_z(0) = 1$. The EDM signal and the effect due to the EQM interaction with field gradients could be observed by measurement of vertical polarization s_y described by Eq. (7b). There only $s_z(t)$ spin component has to be known, and therefore only the formulas for this component will be presented. Moreover, only the solution for $s_z(t)$ valid for $\omega \neq \pm n\omega_o$ will be considered. The special case with $\omega = \pm n\omega_o$ will not be discussed further since it results in the vertical polarization $s_y(t)$ modulated by fast-oscillating harmonic functions, and therefore its time average gives vertical polarization equal zero.

The Taylor expansions about point $\omega_W = 0$ for the solution of Eqs. (7a) and (7c) is applied. For realistic Wien filter fields ω_W is small, therefore this expansion up to first order given by Eqs. (9a) with Φ given by Eq. (9b) and $V(t)$ given by Eq. (10) is used further. Finally the Taylor expansion about point $\Phi = 0$ for $s_z(t)$ and $s_z^2(t)$ (neglecting small terms with $V^2(t) \propto \omega_W$) is performed with the results given in Eqs. (11).

$$s_z(t) = \cos(\omega_s t + \Phi) + V(t)\sin(\omega_s t + \Phi), \tag{9a}$$

$$\Phi = \frac{2\omega_s}{R_0\omega_o}\sum_{n=1}^{\infty}\frac{R_n}{n}\sin n\omega_o t, \tag{9b}$$

$$V(t) = \frac{\omega_W W_0}{2\omega}(\sin(\omega t + \phi) - \sin\phi) - \frac{\omega_W}{2}\sum_{m=1}^{\infty} W_m\left(\frac{\cos((m\omega_o - \omega)t - \phi + \varphi_m) - \cos(\phi - \varphi_m)}{m\omega_o - \omega} + \frac{\cos((m\omega_o + \omega)t + \phi + \varphi_m) - \cos(\phi + \varphi_m)}{m\omega_o + \omega}\right). \tag{10}$$

$$s_z(t) = \sum_{k=0}^{\infty}\cos\left(\omega_s t + k\frac{\pi}{2}\right)\frac{\Phi^k}{k!} + V(t)\sum_{k=0}^{\infty}\sin\left(\omega_s t + k\frac{\pi}{2}\right)\frac{\Phi^k}{k!}, \tag{11a}$$

$$s_z^2(t) = \left(\sum_{k=0}^{\infty}\cos\left(\omega_s t + k\frac{\pi}{2}\right)\frac{\Phi^k}{k!}\right)^2 + V(t)\sum_{k=0}^{\infty}\sin\left(2\omega_s t + k\frac{\pi}{2}\right)\frac{2^k\Phi^k}{k!}. \tag{11b}$$

Time evolution of a vertical polarization induced by EDM is calculated by integration over time the product of magnetic field of Eq. (3a) and $s_z(t)$ given by Eq. (11a) and EQM effect is described by time integral of the product of field gradients Eq. (3b) and $s_z^2(t)$ given by Eq. (11b)

$$s_y^{\mathrm{EDM}}(t) = \omega_e\int_0^t B(t')s_z(t')dt', \tag{12}$$

$$s_y^{\mathrm{EQM}}(t) = \omega_g\int_0^t G(t')s_z^2(t')dt'. \tag{13}$$

Before performing integration the expressions for $s_z(t)$ and $s_z^2(t)$ are reduced to polynomials in $\cos(\omega_o t)$ and $\sin(\omega_o t)$ representing $\cos(m\omega_o t)$ and $\sin(m\omega_o t)$ by Chebyshev polynomials of the first and second kind, respectively, and the multinomial theorem is used. Then it is easy to find out that for the EDM signal for $\omega = n\omega_o \pm \omega_s$ and for the EQM signal for $\omega = n\omega_o \pm 2\omega_s$ the only integrals given by Eq. (14) have terms proportional to time t, where ω_x stands for ω_s in case of Eq. (12) and for $2\omega_s$ in case of Eq. (13)

$$\begin{aligned}\int_0^t \cos^2\omega_x t'\cos^n\omega_o t' dt' &= \int_0^t \cos^2\omega_x t'\sin^n\omega_o t' dt' \\ &= \int_0^t \sin^2\omega_x t'\cos^n\omega_o t' dt' \\ &= \int_0^t \sin^2\omega_x t'\sin^n\omega_o t' dt' \\ &= \frac{1}{2^{n+1}}\binom{n}{n/2}t \\ &\quad + \text{harmonic functions.}\end{aligned} \tag{14}$$

Time average of the terms with harmonic functions in Eq. (14) equals zero and the signals for EDM and EQM are proportional to time. Keeping only the term proportional to time the results of the integration in Eqs. (12) and (13) may be represented by formulas for EDM ($\omega = n\omega_o \pm \omega_s$) given by Eq. (15) and for EQM ($\omega = n\omega_o \pm 2\omega_s$) given by Eq. (16)

$$\begin{gathered}\frac{s_y^{\mathrm{EDM}}}{t} = \pm\frac{\omega_e\omega_W}{8}\sum_{k=0}^{\infty}\frac{1}{k!}\left(\mp\frac{\omega_s}{R_0\omega_o}\right)^k\left(\frac{W_0D_{k,0}\cos\phi}{n\omega_o\pm\omega_s} + \sum_{\substack{m=-\infty\\m\neq 0}}^{\infty}\frac{W_{|m|}D_{k,m}\mathrm{sgn}m\sin(\phi-\varphi_{|m|}\mathrm{sgn}m)}{(m-n)\omega_o\mp\omega_s}\right), \\ D_{0,m} = R_{|n-m|} \quad \text{and} \quad D_{k,m} = \sum_{\substack{j_1=-\infty\\j_1\neq 0}}^{\infty}\frac{R_{|j_1|}}{j_1}\cdots\sum_{\substack{j_k=-\infty\\j_k\neq 0}}^{\infty}\frac{R_{|j_k|}}{j_k}R_{|n-m+j_1+\cdots+j_k|}\mathrm{sgn}\left(\prod_{i=1}^{k}j_i\right),\end{gathered} \tag{15}$$

$$\begin{gathered}\frac{s_y^{\mathrm{EQM}}}{t} = \pm\frac{\omega_g\omega_W}{8}\sum_{k=0}^{\infty}\frac{1}{k!}\left(\mp\frac{2\omega_s}{R_0\omega_o}\right)^k\left(\frac{W_0Q_{k,0}\sin\phi}{n\omega_o\pm 2\omega_s} - \sum_{\substack{m=-\infty\\m\neq 0}}^{\infty}\frac{W_{|m|}Q_{k,m}\mathrm{sgn}m\cos(\phi-\varphi_{|m|}\mathrm{sgn}m)}{(m-n)\omega_o\mp 2\omega_s}\right), \\ Q_{0,m} = (n-m)R_{|n-m|} \quad \text{and} \quad Q_{k,m} = \sum_{\substack{j_1=-\infty\\j_1\neq 0}}^{\infty}\frac{R_{|j_1|}}{j_1}\cdots\sum_{\substack{j_k=-\infty\\j_k\neq 0}}^{\infty}\frac{R_{|j_k|}}{j_k}\left(n-m+\sum_{i=1}^{k}j_i\right)R_{|n-m+j_1+\cdots+j_k|}\mathrm{sgn}\left(\prod_{i=1}^{k}j_i\right).\end{gathered} \tag{16}$$

It is seen that EDM and EQM induced vertical polarization grow-up with time is described by the same parameters, except for the normalization factor depending on the strength of EDM and EQM. Both these effects are separated by Wien filter frequency ω and phase ϕ. Therefore EDM and EQM could be measured independently with only minor changes in Wien filter parameters without affecting particles trajectories.

IV. NUMERICAL RESULTS

The magnitude of vertical polarization build-up with time caused by various effects can be easily estimated using simple modeling of a Wien filter and ring dipoles field distribution. Each of those fields is approximated by a standard analytic midplane field profile for a soft edge 2D dipole given by Eq. (17)

$$b(z) = \frac{1}{2}\left[\tanh\frac{\pi}{2a}(z+L) - \tanh\frac{\pi}{2a}(z-L)\right], \tag{17}$$

where $2L$ is a pole width and $2a$ is a full pole gap.

The numerical calculations are performed for deuteron beam with momentum of 1000 MeV/c. As an example the simple symmetric and periodic lattice consisting of eight identical bending dipoles with $B_0 = 1.5$ T field is considered. The bending magnets are grouped in pairs with a drift space of 1.5 m between them. A drift space between pairs of dipoles is 10 m, 20 m, 10 m, and 20 m. The total length of the lattice is $C = 80$ m and coordinate $z = 0$ is in the middle of drift space between first two pairs of dipoles. The rf Wien filter length is 2 m with the maximum magnetic field $B_{0w} = 0.05$ mT and corresponding maximum electric field $E_{0w} = 7.05$ kV/m. The magnetic field distribution of bending dipoles and Wien filter located at $z_{WF} = C/4$ are shown in Fig. 1.

Since the results for both signs in Wien filter frequency $\omega = n\omega_o \pm \omega_s$ (EDM) and $\omega = n\omega_o \pm 2\omega_s$ (EQM) are very similar, only results for the upper sign will be discussed here.

The indices k, j and m in Eqs. (15) and (16) are running up to infinity. Therefore it is necessary to check convergence of the results as a function of truncation values of corresponding indices $k_{\max}$, $j_{\max}$ and $m_{\max}$ for various Wien filter positions and frequencies. The results of convergence test for the Wien filter located at $z_{WF} = C/2$ and for frequency $\omega = \omega_o + \omega_s$ for EDM and $\omega = \omega_o + 2\omega_s$ for EQM are presented in Fig. 2 for $k_{\max} = 2$ and $j_{\max} = m_{\max}$ varying from 0 to 20 in case of EDM and from 0 to 300 for the EQM signal. These results are normalized to maximum values $s_y^{\mathrm{EDM}}(\max)/t$ and $s_y^{\mathrm{EQM}}(\max)/t$ calculated for $k_{\max} = 2$ and $j_{\max} = m_{\max} = 300$. It is seen that for EDM it is sufficient to use $k = 0$ and small $j_{\max} = m_{\max}$ values to obtain the result with an accuracy better than 1%. In case of the EQM signal the result for $k = 2$ contribute to about 2%, therefore $k = 0, 1$ are sufficient with sums over j and m in Eq. (16) running to approximately 150 to obtain 2% accuracy. Anyhow, in all calculations for which results are presented below the series in Eqs. (15) and (16) were truncated at $k_{\max} = 2$ and $j_{\max} = m_{\max} = 300$.

The resulting signals s_y^{EDM}/t and s_y^{EQM}/t are functions of Wien filter phase ϕ only and this relation depends on the position of the Wien filter in the lattice and on the Wien filter frequency ω i.e. on index n in Eqs. (15) and (16). The behavior of the EDM and EQM signals was investigated in detail for various Wien filter positions $z_{WF}/C = 1/2, 1/3$ and fixed frequency equal to $\omega = \omega_o + \omega_s$ for EDM and $\omega = \omega_o + 2\omega_s$ for EQM, as well as for fixed position $z_{WF}/C = 1/4$ and various frequencies defined by index $n = 1, 2$. The results of these calculations are presented in Fig. 3. For a given Wien filter position and frequency the maximal signal was obtained with a proper choice of the phase ϕ and changing the phase by π the signal change its sign. For any Wien filter position and frequency the function describing the EDM and EQM signals are shifted in phase by $\pi/2$.

For a fixed Wien filter frequency the absolute value of the EDM and EQM signals does not depend on the Wien filter position, however it changes slightly with Wien filter frequency. In Fig. 4 for a fixed phase ($\phi = 0$ for EDM and $\phi = \pi/2$ for EQM) the maximal signal dependence on Wien filter frequency is presented. Changing the index n defining Wien filter frequency the maximal signal changes

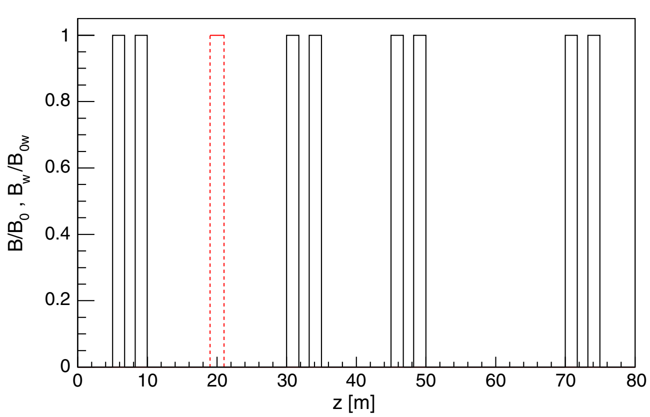

FIG. 1. Dipoles magnetic field (solid line) and Wien filter field (dashed line) distribution in the considered lattice. As an example the Wien filter is located at $z = C/4$, but its position could be changed.

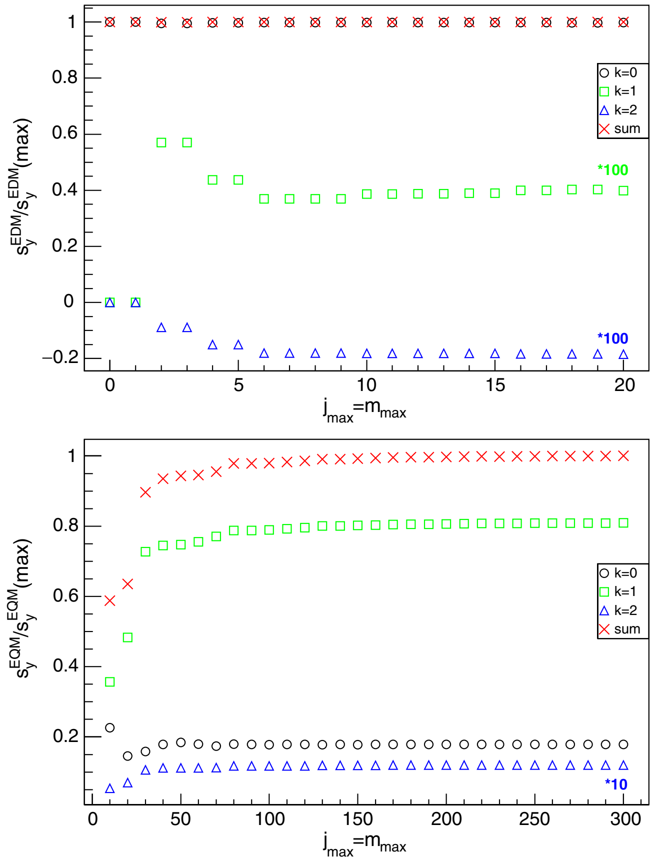

FIG. 2. Convergence of sums in the EDM (top) and EQM (bottom) signals for various $k = 0$, 1, 2 and their sum (correspondingly circles, squares, triangles and crosses) as a function of $j_{\max} = m_{\max}$. For EDM signal results for $k = 1, 2$ are multiplied by 100 and for EQM signal result for $k = 2$ is multiplied by 10 for visualization purposes only.

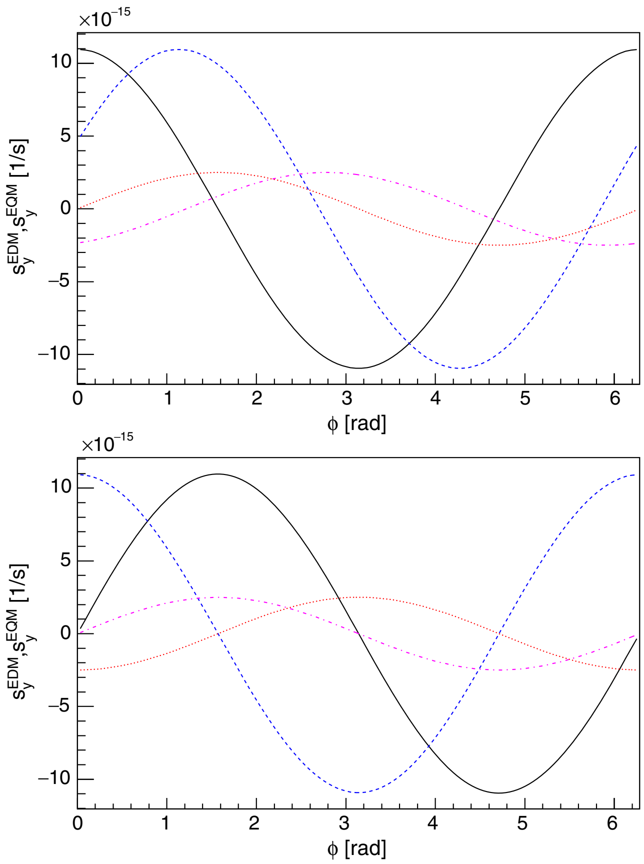

FIG. 3. (Top) The EDM and EQM signals for $n = 1$ and various Wien filter positions $z_{WF} = C/2$ (EDM—solid line, EQM—dotted line) and $z_{WF} = C/3$ (EDM—dashed line and EQM—dash-dotted line). (Bottom) The EDM and EQM signals for $z_{WF} = C/4$ and various Wien filter frequencies for $n = 1$ (EDM—solid line, EQM—dotted line) and $n = 2$ (EDM—dashed line and EQM—dash-dotted line).

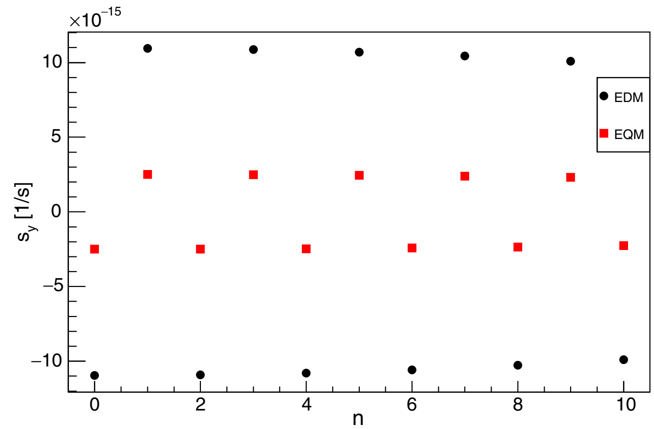

FIG. 4. The EDM (circles for $\omega = n\omega_o + \omega_s$) and EQM (squares for $\omega = n\omega_o + 2\omega_s$) maximal signals as a function of Wien filter frequency (index n). The Wien filter is located at $C/2$ and the Wien filter phase is fixed to $\phi = 0$ for EDM and $\phi = \pi/2$ for EQM.

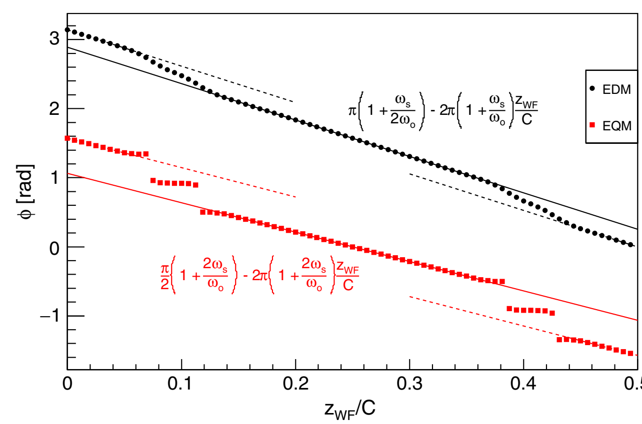

FIG. 5. The Wien filter phase ϕ for maximal EDM (circles for $\omega = \omega_o + \omega_s$) and EQM (squares for $\omega = \omega_o + 2\omega_s$) signals as a function of Wien filter position z_{WF}/C in the lattice. Solid lines with given equations describe the phase evolution between bending dipoles. Dashed lines correspond to phase before the first bending section and behind the second bending section and they are described by similar equations with the offset of $\pm\pi\omega_s/2\omega_o$ in case of the EDM and $\pm\pi\omega_s/\omega_o$ for the EQM signal.

sign but the absolute value remains almost constant. Changing the index n from 0 to 10 the absolute maximum value of the EDM and EQM signals decreases by about 10%.

The presented results demonstrate the importance of the choice of phase of the Wien filter to maximize the signal. As shown in Fig. 3 this phase depends on the Wien filter position and frequency. For the Wien filter located at a drift space between lattice dipoles the phase changes linearly with the Wien filter position and after passing the bending dipoles it increases by $\omega_s/\omega_o \cdot \pi/2$ in the EDM case and by $\omega_s/\omega_o \cdot \pi$ for the EQM signal. The Wien filter phase for maximal signal as a function of Wien filter position is shown in Fig. 5 for the EDM and EQM signals for Wien filter frequency with $n = 1$. Similar dependence can be found for other Wien filter frequencies.

Similar analysis was performed also for COSY synchrotron, where the precursor EDM experiment is planned [10]. Generally all the features discussed for simple lattice holds also in case of COSY. The only major difference is that expected EQM signal is by factor about 14 smaller than the goal EDM signal of 10^{-29} e cm.

V. CONCLUSION

The vertical spin component induced by EDM in a storage ring containing a Wien filter was considered in a simple but realistic model. For the first time the influence of the EQM interaction with field gradients on the vertical spin component was analyzed. The resonant Wien filter frequencies for EDM and EQM interacting with dipoles gradients were identified. It was shown that the EDM and EQM effects are separated by different Wien filter resonant frequency and phase.

It was shown that the strength of the EDM and EQM signals have similar order of magnitude (in the considered case of a simple lattice $s_y^{\mathrm{EDM}}/s_y^{\mathrm{EQM}} = 4.4$). Therefore the EQM—gradients interaction could be used for studying the magnitude and sources of systematic uncertainties in EDM measurements. In the storage ring designed to reach 10^{-29} e cm accuracy for EDM, reproducing deuteron EQM value known with precision of 10^{-3} should pin down the systematic uncertainty to 10^{-32} e cm. This would allow us to reach the region of standard model prediction for the EDM value. Therefore it is proposed to perform the final EDM measurement with quasisimultaneous monitoring of the systematic uncertainties with minor modification of the experimental conditions. The measurement could be performed by setting in subsequent machine cycles the Wien filter operating conditions maximizing the EDM–ad EQM-induced effects. In this way the time variation of the experimental conditions during long measurements could be also monitored. One should consider measurement at several Wien filter frequencies since both EDM and EQM signals change sign by changing Wien filter frequency by orbital frequency. It would allow an additional cross-check of the estimated systematic uncertainties.

The results of the present analysis demonstrate that the measurements of the MDM and EQM interaction with field gradients could be very useful for studying the systematics in high precision EDM measurements. While here only consequences for one specific method of EDM measurement were considered, similar studies should be performed for other proposed methods.

While the presented model could quite well predict the general features of the storage ring EDM measurement including EQM interaction with field gradients, a more advanced calculations with the tracking codes are necessary. Therefore it is called for the extension of the existing tracking codes by including MDM and EQM interaction with field gradients. The presented analytic solution could be used to benchmark the tracking codes, when they have been supplemented by effects related with the presence of field gradients.

ACKNOWLEDGMENTS

This paper was motivated by ongoing work towards an EDM measurement with a storage ring performed by the JEDI (Jülich Electric Dipole Moment Investigations) collaboration. The author would like to thank the whole JEDI collaboration for the encouragement to work on the presented problem and for stimulative discussions on the subject.

[1] F. J. M. Farley, K. Jungmann, J. P. Miller, W. M. Morse, Y. F. Orlov, B. L. Roberts, Y. K. Semertzidis, A. Silenko, and E. J. Stephenson, New Method of Measuring Electric Dipole Moments in Storage Rings, Phys. Rev. Lett. **93**, 052001 (2004).

[2] Y. K. Semertzidis, The status of the storage ring EDM experiment, AIP Conf. Proc. **1182**, 730 (2009).

[3] V. Anastassopoulos *et al.*, A storage ring experiment to detect a proton electric dipole moment, Rev. Sci. Instrum. **87**, 115116 (2016).

[4] Y. Senichev, A. Lehrach, B. Lorentz, R. Maier, S. Andrianov, A. Ivanov, S. Chekmenev, M. Berz, and E. Valetov", Quasi-frozen spin method for EDM deuteron search, in *Proceedings of IPAC15, Richmond, Virginia, USA* (2015), p. 213.

[5] Y. Senichev, S. Andrianov, A. Ivanov, S. Chekmenev, M. Berz, and E. Valetov, Investigation of lattice for deuteron EDM ring, in *Proceedings of ICAP 2015, Shanghai, China* (2015), p. 17.

[6] Y. F. Orlov, W. M. Morse, and Y. K. Semertzidis, Resonance Method of Electric-Dipole-Moment Measurements in Storage Rings, Phys. Rev. Lett. **96**, 214802 (2006).

[7] W. M. Morse, Y. F. Orlov, and Y. K. Semertzidis, rf Wien filter in an electric dipole moment storage ring: The "partially frozen spin" effect, Phys. Rev. ST Accel. Beams **16**, 114001 (2013).

[8] JEDI Collaboration, web page: http://collaborations.fzjuelich.de/ikp/jedi.

[9] A. Lehrach, B. Lorentz, W. Morse, N. Nikolaev, and F. Rathmann, Precursor experiments to search for permanent electric dipole moments (EDMs) of protons and deuterons at COSY, in *Proc. of XIV Advanced Research Workshop on High Energy Spin Physics (DSPIN'11)* (2011) p. 287.

[10] F. Rathmann, A. Saleev, and N. N. Nikolaev, The search for electric dipole moments of light ions in storage rings, J. Phys. Conf. Ser. **447**, 012011 (2013).

[11] S. Chekmenev, Estimation of systematic errors for deuteron electric dipole moment search at COSY, Int. J. Mod. Phys. Conf. Ser. **40**, 1660099 (2016).

[12] S. Chekmenev, Ph.D. thesis, RWTH Aachen U., 2016, http://collaborations.fzjuelich.de/ikp/jedi/public_files/theses/Thesis_Chekmenev.pdf.

[13] A. Saleev *et al.*, Spin tune mapping as a novel tool to probe the spin dynamics in storage rings, Phys. Rev. Accel. Beams **20**, 072801 (2017).

[14] P. J. Mohr, B. N. Taylor, and D. B. Newell, CODATA recommended values of the fundamental physical constants: 2010, Rev. Mod. Phys. **84**, 1527 (2012).

[15] D. M. Bishop and L. M. Cheung, Quadrupole moment of the deuteron from a precise calculation of the electric field gradient in D_2, Phys. Rev. A **20**, 381 (1979).

[16] T. E. O. Ericson and M. Rosa-Clot, The deuteron asymptotic D-state as a probe of the nucleon-nucleon force, Nucl. Phys. **A405**, 497 (1983).

[17] V. Bargmann, L. Michel, and V. L. Telegdi, Precession of the Polarization of Particles Moving in a Homogeneous Electromagnetic Field, Phys. Rev. Lett. **2**, 435 (1959).

[18] R. H. Good, Classical equations of motion for a polarized particle in an electromagnetic field, Phys. Rev. **125**, 2112 (1962).

[19] A. I. Solomon, Motion of a particle with spin in a nonhomogeneous electromagnetic field, Nuovo Cimento **26**, 1320 (1962).

[20] P. Nyborg, Approximate relativistic equations of motion for an extended charged particle in an inhomogeneous external electromagnetic field, Nuovo Cimento **31**, 1209 (1964).

[21] T. Fukuyama and A. J. Silenko, Derivation of generalized Thomas-Bargmann-Michel-Telegdi equation for a particle with electric dipole moment, Int. J. Mod. Phys. A **28**, 1350147 (2013).

[22] A. J. Silenko, Phys. Atom. Nucl. **64**, 983 (2001).

[23] A. Y. Silenko, The motion of particle spin in a nonuniform electromagnetic field, J. Exp. Theor. Phys. **96**, 775 (2003).

[24] T. O. Niinikoski and R. Rossmanith, Self-polarization of protons in storage rings, Nucl. Instrum. Methods Phys. Res., Sect. A **255**, 460 (1987).

[25] Wolfram Research Inc., Mathematica 10.0 (2016).

[26] A. Magiera, talks at JEDI Collaboration Meetings, Jülich, 2015/2016.